Appendix B

Derivation of the Approximations to the Pilot Visibility Probabilities

In this Appendix, we drive the approximations $\tilde{\theta}_{ij}(x,y)$ of the pilot visibility probabilities. The notation used in this Appendix is the same as what has been introduced earlier in this specification. Consider a grid point $(x,y) \in A$. As in the above referenced related application Serial No.___________, entitled, "Geolocation Estimation Method For CDMA Terminals Based On Pilot Strength Measurements", the RF power received by the mobile unit 20 from sector j of base station i is modeled by the expression

Sub a9

$$R_{ij}(x,y) = T_{ij} G_{ij}(x,y) L_{ij}(x,y) F_{ij}(x,y) M_{ij}(x,y) \quad \text{(B1)}$$

where T_{ij} is the transmit power associated with the sector, $G_{ij}(x,y)$ is the antenna gain for the sector along the direction pointing towards the location $(x,y) \in A$, $L_{ij}(x,y)$ is the distance loss between the base station associated with the sector and the location $(x,y) \in A$, $F_{ij}(x,y)$ is the shadow fading factor and $M_{ij}(x,y)$ is the measurement noise factor, all in absolute, not dB, units. The measurement noise factor is meant to include the effects of fast fading (e.g., Rayleigh/Rician) as well as inaccuracies in the measurement process. If γ denotes the fraction of T_{ij} that is used for the pilot channel, then $\gamma R_{ij}(x,y)$ is the pilot channel power received by the mobile unit 20 when it is located at $(x,y) \in A$.

The model assumes that the distance loss $L_{ij}(x,y)$ can be expressed as

$$L_{ij}(x,y) = C_p [\, d_{ij}(x,y) \,]^{-\alpha} \quad \text{(B2)}$$

where $d_{ij}(x,y)$ is the distance between the base station associated with the sector and the location $(x,y) \in A$, and C_p and α are constants. Typically, C_p takes a value in the range 10^{-15} to 10^{-10} and α is between 3 and 5 when $d_{ij}(x,y)$ is expressed in miles. The two parameters, C_p and α, moreover are environment specific.

The shadow fading factor, $F_{ij}(x,y)$, models the impact of terrain and large structures (e.g., buildings) on signal propagation which create deviations around the signal

attenuation predicted by the deterministic path loss factor. We assume that $F_{ij}(x,y)$ is lognormally distributed and thus can be written as

$$F_{ij}(x,y) = e^{-\frac{1}{2}\sigma_\phi^2} e^{\varphi_{ij}(x,y)} \tag{B3}$$

where $\phi_{ij}(x,y)$ is a zero mean Gaussian random variable with standard deviation σ_ϕ.

The measurement noise factor, $M_{ij}(x,y)$ is also assumed to have a lognormal distribution so that we can write

$$M_{ij}(x,y) = e^{-\frac{1}{2}\sigma_\mu^2} e^{\mu_{ij}(x,y)} \tag{B4}$$

where $\mu_{ij}(x,y)$ is a zero mean Gaussian random variable with standard deviation σ_μ.

Assume now that the pilot strength measurement carried out and reported to the base station by the mobile unit 20 is the Ec/Io value of the corresponding pilot channel signal. This value is the ratio of the pilot channel power from the concerned sector received by the mobile unit 20 to the total power received by the mobile unit including thermal noise, and possibly external interference.

By letting $P_{ij}(x,y)$ denote the strength of the pilot channel associated with the sector as measured by the mobile unit 20 located at $(x,y) \in A$, it follows that

$$P_{ij}(x,y) = \frac{\gamma R_{ij}(x,y)}{N_0 + \sum_{kl \in K} R_{kl}(x,y)} \tag{B5}$$

where N_0 denotes thermal noise and external interference and (as before) *kl* is shorthand notation for the pilot associated with sector *l* of base station *k*.

For convenience, we define

$$C_{ij}(x,y) = T_{ij} G_{ij}(x,y) L_{ij}(x,y). \tag{B6}$$

Observe that the expected value of $R_{ij}(x,y)$ is equal to $C_{ij}(x,y)$. It is implicitly assumed that the shadow fading and measurement noise factors are uncorrelated. We approximate $P_{ij}(x,y)$ (B5) by the following expression

$$Z_{ij}(x,y) = \frac{\gamma R_{ij}(x,y)}{N_0 + R_{ij}(x,y) + \sum\limits_{\substack{kl\in K \\ kl\neq ij}} C_{kl}(x,y)} \qquad \text{(B7)}$$

The difference between (B7) and (B5) is that for $kl \neq ij$, the $R_{kl}(x,y)$ quantities have been replaced by their mean values. Using (B3) and (B4), we can write

$$Z_{ij}(x,y) = \frac{be^{\xi}}{a + ce^{\xi}} \qquad \text{(B8)}$$

where

$$a = e^{\frac{1}{2}(\sigma_\phi^2 + \sigma_\mu^2)}[\, N_0 + \sum_{\substack{kl\in K \\ kl\neq ij}} C_{kl}(x,y)\,]$$

$$b = \gamma C_{ij}(x,y)$$

$$c = C_{ij}(x,y)$$

$$\xi = \phi_{ij}(x,y) + \mu_{ij}(x,y)\,.$$

The probability that the mobile unit receives pilot ij with a signal strength in excess of T is

$$\begin{aligned} \theta_{ij}(x,y) &= \Pr[\, P_{ij}(x,y) > T\,] \\ &\cong \Pr[\, Z_{ij}(x,y) > T\,] \\ &\equiv \widetilde{\theta}_{ij}(x,y)\,. \end{aligned}$$

It can be easily shown that

$$\widetilde{\theta}_{ij}(x,y) = Q\left[-\frac{Ln\left(\frac{b}{a}T^{-1} - \frac{c}{a}\right)}{\sigma} \right] \qquad \text{(B9)}$$

where $\sigma = (\sigma_\phi^2 + \sigma_\mu^2)^{1/2}$ and $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{1}{2}t^2}\, dt$.

Note that the expression for $\widetilde{\theta}_{ij}(x,y)$ depends on the following unknown parameters: σ_ϕ^2, σ_μ^2, N_0, C_p and α. As in the above noted related application entitled, "Geolocation Estimation ....", in the present invention, pilot strength measurements are first

carried out along a small number of representative routes in the service area. The measurements so obtained are then used to select values of σ_{ϕ}^{2}, σ_{μ}^{2}, N_0, C_p and α which provide the best fit between the predicted pilot visibility probabilities, using equation (B9), and the observed pilot visibility probabilities